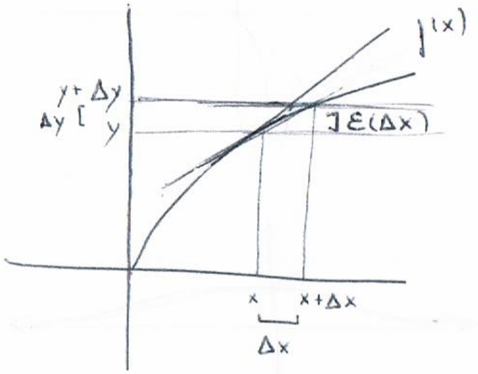
② Demostrar que el error cometido $\varepsilon(\Delta x)$ x la aprox lineal $(f'(x)\cdot\Delta x)$ del incremento real Δy de la función $\to 0$ en comparación con Δx (cuando $\Delta x \to 0$). Significado geométrico + figura.



$f'(x) \simeq \dfrac{\Delta y}{\Delta x}$

$\Delta y \simeq \Delta x \cdot f'(x)$

$\Delta y = \Delta x \cdot f'(x) + \varepsilon(\Delta x)$

$\Delta y - \Delta x \cdot f'(x) = \varepsilon(\Delta x)$

$\dfrac{\Delta y - \Delta x \cdot f'(x)}{\Delta x} = \dfrac{\varepsilon(\Delta x)}{\Delta x}$

~~$f(x+\Delta x) \simeq f(x)\, f'(x) + \varepsilon(\Delta x)$~~

1° Def. derivada $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} \simeq f'(x)$

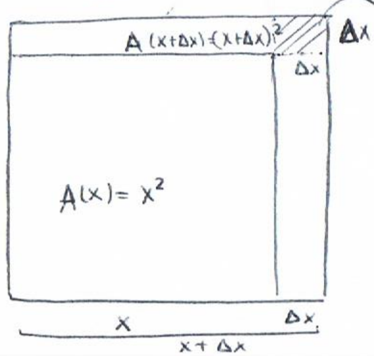
2° $\varepsilon(\Delta x)$ $\quad f(x+\Delta x) - f(x) - f'(x)\cdot\Delta x = \varepsilon(\Delta x)$

3° $/\Delta x$ $\quad \dfrac{f(x+\Delta x) - f(x) - f'(x)\cdot\Delta x}{\Delta x}$ $\quad \lim_{\Delta x\to 0} \dfrac{\varepsilon(\Delta x)}{\Delta x}$

4° lim $\lim_{\Delta x \to 0}$

$f'(x) - f'(x) = 0 = \boxed{\dfrac{\varepsilon(\Delta x)}{\Delta x}}$

luego $\varepsilon(\Delta x)$ es un infinitésimo de orden mayor que Δx.
(No sólo $\varepsilon(\Delta x) \to 0$ cuando $\Delta x \to 0$, sino también $\dfrac{\varepsilon(\Delta x)}{\Delta x} \to 0$ cuando $\Delta x \to 0$).



aprox lineal $f(x+\Delta x) = f'(x)\cdot\Delta x + f(x)$

$A(x+\Delta x) - A(x) = A'(x)\cdot\Delta x + A(x) - A x$

$= 2x\cdot\Delta x + \cancel{A(x)} - \cancel{A(x)}$

En este caso el error es $[\varepsilon(\Delta x)]^2 = (\Delta x)^2$

$\displaystyle\int \frac{1}{x^2-x-2}\,dx = \int \frac{1/3}{x-2}\,dx + \int \frac{-1/3}{x+1}\,dx =$

$= \frac{1}{3}\ln|x-2| - \frac{1}{3}\ln|x+1| + k$

$x^2 - x - 2 = 0$

$x = \dfrac{+1 \pm \sqrt{1-4\cdot(-2)}}{2} = \dfrac{1\pm 3}{2}$ $\quad x_1 = 2,\ x_2 = -1$

$\dfrac{1}{x^2-x-2} = \dfrac{A}{(x-2)} + \dfrac{B}{(x+1)} = \dfrac{Ax+A+Bx-2B}{x^2-x-2} =$

$= \begin{cases} A+B = 0 \\ A-2B = 1 \end{cases}$ $\quad A = -B$, $\ -B-2B = 1$, $\ B = -1/3$, $\ A = 1/3$

$\displaystyle\int \cos^2 x\,dx = \int \frac{\cos 2x + 1}{2} =$

$= \frac{1}{2}\int \cos(2x) + \frac{1}{2}\int dx =$

$= \frac{1}{2}\cdot\frac{1}{2}\int \cos(2x)\cdot 2\,dx + \frac{1}{2}x =$

$= \frac{1}{4}\,\text{sen}(2x) + \frac{1}{2}x + k$

$\boxed{\begin{array}{l}\cos^2 x = 1 - \text{sen}^2 x \\ \cos 2x = \cos^2 x - \text{sen}^2 x\end{array}}$

~~$\text{sen}\ \cos^2 x = \cos^2 x + \text{sen}^2 x$~~

$\text{sen}\,2x = 2\,\text{sen}\,x\cos x$

$\cos^2 x = 1 - (\cos^2 x - \cos 2x)$

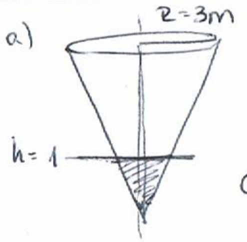
$\cos^2 x = \cos 2x + \text{sen}^2 x$

$\cos^2 x = \cos 2x + 1 - \cos^2 x$

$2\cos^2 x = \cos 2x + 1$

$\boxed{\cos^2 x = \dfrac{\cos 2x + 1}{2}}$

③ a)



$h = 8\,m$

$\dfrac{dV}{dt} = 0'11\ m^3/min$

Cuando $h = 1m \to$ se extrae a $\dfrac{dV}{dt} = -0'1\ m^3/min$ ¿$\dfrac{dh}{dt}$ cuando $h = 7m$?

$\dfrac{R}{h} = \dfrac{3}{8}$

$\dfrac{dV}{dt} = \dfrac{dV}{dh}\cdot\dfrac{dh}{dt}$

$V = \frac{1}{3}\pi R^2\cdot h = \frac{1}{3}\pi\,\dfrac{9h^3}{8^2}$

$\dfrac{dV}{dh} = \cancel{\frac{1}{3}\pi R^2}\ \dfrac{\pi 9h^2}{64}$

$\dfrac{dV}{dt} = 0'11 - 0'1 = \pi\,\dfrac{9h^2}{64}\cdot\dfrac{dh}{dt}$

$\dfrac{dh}{dt} = 0'01 / \pi\,\dfrac{9h^2}{64} = 0'1/\pi\,\dfrac{9\cdot 7^2}{64} =$

$= 4'62\cdot 10^{-4}$ m/s